

Metric Dimension of Amalgamation of Regular Graphs

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Abstract

A set of vertices S resolves a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S . The metric dimension of G is the minimum cardinality of a resolving set of G .

Let $\{G_1, G_2, \dots, G_n\}$ be a finite collection of graphs and each G_i has a fixed vertex v_{0_i} or a fixed edge e_{0_i} called a terminal vertex or edge, respectively. The *vertex-amalgamation* of G_1, G_2, \dots, G_n , denoted by $Vertex - Amal\{G_i; v_{0_i}\}$, is formed by taking all the G_i 's and identifying their terminal vertices. Similarly, the *edge-amalgamation* of G_1, G_2, \dots, G_n , denoted by $Edge - Amal\{G_i; e_{0_i}\}$, is formed by taking all the G_i 's and identifying their terminal edges.

Here we study the metric dimensions of vertex-amalgamation and edge-amalgamation for finite collection of regular graphs: complete graphs and prisms.

Key words: graph distance; resolving set; metric dimension; amalgamation; complete graphs; prisms

1 Introduction

In this paper we consider finite, simple, and connected graphs. The vertex and edge sets of a graph G are denoted by $V(G)$ and $E(G)$, respectively.

The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G . For an ordered set $S = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$, we refer to the k -vector $r(v|S) = (d(v, v_1), d(v, v_2), \dots, d(v, v_k))$ as the *(metric) representation of v with respect to S* . The set S is called a *resolving set* for G if $r(u|S) = r(v|S)$ implies that $u = v$ for all $u, v \in G$. In a graph G ,

a resolving set with minimum cardinality is called a *basis* for G . The *metric dimension*, $\dim(G)$, is the number of vertices in a basis for G .

The metric dimension problem was first introduced in 1975 by Slater [28], and independently by Harary and Melter [11] in 1976; however the problem for hypercube was studied (and solved asymptotically) much earlier in 1963 by Erdős and Rényi [6]. In general, it is difficult to obtain a basis and metric dimension for arbitrary graph. Garey and Johnson [9], and also Khuller *et al.* [19], showed that determining the metric dimension of an arbitrary graph is an NP-complete problem. The problem is still NP-complete even if we consider some specific families of graphs, such as bipartite graphs [21] or planar graphs [5]. Thus research in this area are then constrained towards: characterizing graphs with particular metric dimensions, determining metric dimensions of particular graphs, and constructing algorithm that best approximate metric dimensions.

Until today, only graphs of order n with metric dimension 1 (the paths), $n - 3$, $n - 2$, and $n - 1$ (the complete graphs) have been characterized [4,12,17]. On the other hand, researchers have determined metric dimensions for many particular classes of graphs. There are also some results of metric dimensions of graphs resulting from graph operations; for instance: Cartesian product graphs [22,19,3], join product graphs [1,2], strong product [24], corona product graphs [30,15], lexicographic product graphs [26], hierarchical product graphs [7], line graphs [18,8], and permutation graphs [10].

In this paper, we study metric dimension of graphs resulting from another type of graph operations, i.e., vertex-amalgamation and edge-amalgamation. Let $\{G_1, G_2, \dots, G_n\}$ be a finite collection of graphs and each *block* G_i has a fixed vertex v_{0_i} or a fixed edge e_{0_i} called a *terminal vertex* or *edge*, respectively. The *vertex-amalgamation* of G_1, G_2, \dots, G_n , denoted by $Vertex - Amal\{G_i; v_{0_i}\}$, is formed by taking all the G_i 's and identifying their terminal vertices. Similarly, the *edge-amalgamation* of G_1, G_2, \dots, G_n , denoted by $Edge - Amal\{G_i; e_{0_i}\}$, is formed by taking all the G_i 's and identifying their terminal edges.

Previous study of amalgamation of graphs has been done for vertex-amalgamation of two arbitrary graphs [23], vertex-amalgamation of cycles [13,14], and edge-amalgamation of cycles [27]. Poisson and Zhang studied vertex-amalgamation of two nontrivial connected graphs G_1, G_2 and provide a lower bound as follow.

Theorem 1. [23] *Let G be the vertex-amalgamation of nontrivial connected graphs G_1 and G_2 . Then*

$$\dim(G) \geq \dim(G_1) + \dim(G_2) - 2.$$

Other known results are vertex-amalgamation and edge-amalgamation of cy-

cles. We denote by C_n the cycle of order n .

Theorem 2. [14,27] *Let $\{C_{c_1}, C_{c_2}, \dots, C_{c_n}\}$ be a collection of n cycles with n_e cycles of even order. Suppose that G is the vertex-amalgamation of $C_{c_1}, C_{c_2}, \dots, C_{c_n}$ and H is the edge-amalgamation of $C_{c_1}, C_{c_2}, \dots, C_{c_n}$. Then*

$$\dim(G) = \begin{cases} \sum_{i=1}^n \dim(C_{c_i}) - n & , n_e = 0, \\ \sum_{i=1}^n \dim(C_{c_i}) - n + n_e - 1 & , n_e \geq 1 \end{cases}$$

and

$$\sum_{i=1}^n \dim(C_{c_i}) - n - 2 \leq \dim(H) \leq \sum_{i=1}^n \dim(C_{c_i}) - n.$$

The previous theorem provided the metric dimensions of vertex and edge amalgamation of connected 2-regular graphs. In the next section, we shall consider metric dimensions of vertex-amalgamation and edge-amalgamation of other connected regular graphs: complete graphs and prisms.

2 Main Results

2.1 Metric Dimension of Amalgamation of Complete Graphs

Two vertices u and v of a graph G is defined in [25] to be *distance similar* if $d(u, x) = d(v, x)$ for all $x \in V(G) - \{u, v\}$. Certainly, distance similarity is an equivalence relation in $V(G)$. The following observation is useful.

Observation 1. [25] *Let G be a graph and let V_1, V_2, \dots, V_k be the k distinct distance-similar equivalence classes of $V(G)$. If W is a resolving set of G , then W contains at least $|V_i| - 1$ vertices from each equivalence class V_i for all i and so $\dim(G) \geq |V(G)| - k$.*

Let K_k be a complete graph of order k . It is obvious that K_k is a $(k-1)$ -regular graph. Consider vertex-amalgamation of $\{K_{k_1}, K_{k_2}, \dots, K_{k_n}\}$ a collection of n complete graphs, where k_i is of an increasing order. We denote by $v_1^i, v_2^i, \dots, v_{k_i}^i$ the vertices in the block K_{k_i} , $c = v_{k_i}^i$ the terminal vertex, and K_{k_i-1} the subgraph obtained by deleting c from the block K_{k_i} .

Theorem 3. *Let $\{K_{k_1}, K_{k_2}, \dots, K_{k_n}\}$ be a collection of n complete graphs with n_2 complete graphs of order 2. If G is the vertex-amalgamation of K_{k_1}, \dots, K_{k_n} then*

$$\dim(G) = \begin{cases} \sum_{i=1}^n \dim(K_{k_i}) - n + n_2 - 1 & , n_2 \geq 2, \\ \sum_{i=1}^n \dim(K_{k_i}) - n & , \text{otherwise} \end{cases}$$

Proof. For $n_2 \geq 2$, let $V_c = \{c\}$, $V_0 = \{v_1^1, v_1^2, \dots, v_1^{k_{n_2}}\}$ and $V_i = V(K_{k_{n_2+i-1}})$, $i = 1, \dots, n - n_2$. Clearly, $V_c, V_0, V_1, V_2, \dots, V_{n-n_2}$ are distance-similar equivalence classes of $V(G)$. By Observation 1, a resolving set of G contains at least $|V_i| - 1$ vertices from each equivalence class V_i and so $\dim(G) \geq 0 + (n_2 - 1) + \sum_{i=1}^{n-n_2} ((n_2 + i - 1) - 1) = \sum_{i=1}^n \dim(K_{k_i}) - n + n_2 - 1$. Consider $S = \{v_1^i | i = 1, \dots, n_2 - 1\} \cup \{v_1^i, v_2^i, \dots, v_{k_i-2}^i | i = n_2 + 1, \dots, n\}$. Thus $r(c|S) = (1, \dots, 1)$, $r(v_1^{n_2}|S) = (2, \dots, 2)$, and the coordinates in $r(v_{k_i-1}^i|S)$ are 1 for those correspond with vertices in block K_{k_i} and 2 otherwise. This results in S being a resolving set and $\dim(G) \leq \sum_{i=1}^n \dim(K_{k_i}) - n + n_2 - 1$.

For $n_2 = 1$, let $V_c = \{c\}$, $V_0 = v_1^1$, and $V_i = V(K_{k_{n_2+i-1}})$, $i = 1, \dots, n - n_2$, then we have $V_c, V_0, V_1, \dots, V_{n-n_2}$ to be distance-similar equivalence classes of $V(G)$. By Observation 1, a resolving set of G contains at least $(k_{n_2+i-1}) - 1 = k_{n_2+i} - 2$ vertices from each V_i ; thus $\dim(G) \geq \sum_{i=1}^n \dim(K_{k_i}) - n$. Choose $S = \{v_1^i, v_2^i, \dots, v_{k_i-2}^i | i = n_2 + 1, \dots, n\}$, then $r(c|S) = (1, \dots, 1)$, $r(v_1^1|S) = (2, \dots, 2)$ and the coordinates in $r(v_{k_i-1}^i|S)$ are 1 for those correspond with vertices in block K_{k_i} and 2 otherwise. Therefore S resolves G and so $\dim(G) \leq \sum_{i=1}^n \dim(K_{k_i}) - n$.

For $n_2 = 0$, let $V_c = \{c\}$ and $V_i = V(K_{k_{n_2+i-1}})$, $i = 1, \dots, n$ and so V_c, V_1, \dots, V_n are distance-similar equivalence classes of $V(G)$. Thus, $\dim(G) \leq \sum_{i=1}^n k_{n_2+i} - 2$. Now define $S = \{v_1^i, v_2^i, \dots, v_{k_i-2}^i | i = 1, \dots, n\}$, then $r(c|S) = (1, \dots, 1)$ and the coordinates in $r(v_{k_i-1}^i|S)$ are 1 for those correspond with vertices in block K_{k_i} and 2 otherwise. Therefore S resolves G which leads to $\dim(G) \leq \sum_{i=1}^n \dim(K_{k_i}) - n$ and this completes the proof. \square

Consider edge-amalgamation of $\{K_{k_1}, K_{k_2}, \dots, K_{k_n}\}$ a collection of n complete graphs, where k_i is of an increasing order. We denote by $v_1^i, v_2^i, \dots, v_{k_i-2}^i$ the vertices in the block K_{k_i} , $c_1 c_2 = v_{k_i-1}^i v_{k_i}^i$ the terminal edge, and K_{k_i-2} the subgraph obtained by deleting $c_1 c_2$ from the block K_{k_i} .

Theorem 4. *Let $\{K_{k_1}, K_{k_2}, \dots, K_{k_n}\}$ be a collection of n complete graphs with n_3 complete graphs of order 3. If G is the edge-amalgamation of K_{k_1}, \dots, K_{k_n} then*

$$\dim(G) = \begin{cases} \sum_{i=1}^n \dim(K_{k_i}) - 2n + 1, & n_3 = 0, \\ \sum_{i=1}^n \dim(K_{k_i}) - 2n + 2, & n_3 = 1 \text{ and } n = 2, \\ \sum_{i=1}^n \dim(K_{k_i}) - 2n + n_3, & \text{otherwise.} \end{cases}$$

Proof. For $n_3 = 0$, let $V_0 = \{c_1, c_2\}$ and $V_i = V(K_{k_i-2})$, $i = 1, 2, \dots, n$. V_0, V_1, \dots, V_n are distance similar equivalence classes of $V(G)$. By Observation 1, a resolving set of G contains at least $|V_i| - 1$ vertices from each equivalence class V_i and so $\dim(G) \geq \sum_{i=1}^n \dim(K_{k_i}) - 2n + 1$. Define a set $S = \{c_1\} \cup \{v_1^i, v_2^i, \dots, v_{k_i-3}^i | i = 1, 2, \dots, n\}$, then $r(c_2|S) = (1, \dots, 1)$ and the co-

ordinates in $r(v_{k_i-2}^i|S)$ are 1 for those correspond with c_1 and vertices in block K_{k_i} and 2 otherwise. Thus S resolves G and $\dim(G) \leq \sum_{i=1}^n \dim(K_{k_i}) - 2n + 1$.

For $n_3 = 1$ and $n = 2$, we have $\{v_1^1\}$, K_{k_2-2} , and $\{c_1, c_2\}$ as distance similar equivalence classes of $V(G)$. By Observation 1, a resolving set of G contains at least $|K_{k_2-2}| - 1$ vertices of K_{k_2-2} and 1 vertices of $\{c_1, c_2\}$ or $\dim(G) \geq (k_2 - 3) + 1 = k_2 - 2$. Assume R is a resolving set with cardinality $k_2 - 2$, thus there exist $a \in K_{k_2-2}$ and $b \in \{c_1, c_2\}$ which are not contained in R . In this case $r(a|R) = (1, 1, \dots, 1) = r(b|R)$, a contradiction. Therefore $\dim(G) \geq k_2 - 1$. Let $S = \{v_1^1\} \cup \{v_1^2, v_2^2, \dots, v_{k_2-3}^2\} \cup \{c_1\}$. Thus we have $r(c_2|S) = (1, \dots, 1)$, $r(v_{k_2-2}^2|S) = (2, 1, 1, \dots, 1)$ and so $\dim(G) \leq 1 + (k_2 - 3) + 1 = k_2 - 1$. Therefore $\dim(G) = k_2 - 1 = \sum_{i=1}^n \dim(K_{k_i}) - 2n + 2$.

For $n_3 = n$, the sets $\{c_1, c_2\}$ and $\{v_1^1, v_1^2, \dots, v_1^{n_3}\}$ are distance similar equivalence classes of $V(G)$. By applying Observation 1, we have $\dim(G) \geq 1 + (n_3 - 1) = n_3$. Let $S = \{c_1\} \cup \{v_1^1, v_1^2, \dots, v_1^{n_3-1}\}$, and so $r(c_2|S) = (1, 1, \dots, 1)$, and $r(v_1^{n_3}|S) = (1, 2, 2, \dots, 2)$. Thus $\dim(G) \leq n_3$ and we have $\dim(G) = n_3 = \sum_{i=1}^n \dim(K_{k_i}) - 2n + n_3$.

For the rest of the cases, let $V_c = \{c_1, c_2\}$, $V_0 = \{v_1^1, v_1^2, \dots, v_1^{n_3}\}$, $V_i = V(K_{k_{n_3+i}-2})$, $i = 1, 2, \dots, n - n_3$. We can see that $V_c, V_0, V_1, \dots, V_{n-n_3}$ are distance similar equivalence classes of $V(G)$. By using Observation 1, $\dim(G) \geq 1 + (n_3 - 1) + \sum_{i=n_3+1}^n ((k_i - 2) - 1) = \sum_{i=n_3+1}^n (k_i - 3) + n_3$. Choose $S = \{c_1\} \cup V_0 \cup \{v_1^{n_3+i}, v_2^{n_3+i}, \dots, v_{k_{n_3+i}-3}^{n_3+i} | i = 1, 2, \dots, n - n_3\}$. Then we have $r(c_2|S) = (1, \dots, 1)$, $r(v_1^{n_3}|S) = (1, 2, \dots, 2)$ and the coordinates in $r(v_{k_{n_3+i}-2}^{n_3+i}|S)$ are 1 for those correspond with c_1 and vertices in block K_{n_3+i} and 2 otherwise. Therefore $\dim(G) \leq \sum_{i=n_3+1}^n (k_i - 3) + n_3$ and, consequently, $\dim(G) = \sum_{i=n_3+1}^n (k_i - 3) + n_3 = \sum_{i=1}^n \dim(K_{k_i}) - 2n + n_3$. \square

2.2 Metric Dimension of Amalgamation of Prisms

For $n \geq 3$, a prism $Pr_n = C_n \times P_2$ is a 3-regular graphs of order $2n$. Let $V(Pr_n) = \{u_1, \dots, u_n, v_1, \dots, v_n\}$ and $E(Pr_n) = \{u_i v_i, i = 1, \dots, n\} \cup \{u_i u_{i+1} | i = 1, \dots, n\} \cup \{u_n u_1\} \cup \{v_i v_{i+1} | i = 1, \dots, n\} \cup \{v_n v_1\}$. Consider vertex-amalgamation of $\{Pr_{p_1}, Pr_{p_2}, \dots, Pr_{p_n}\}$ a collection of n prisms. We denote by $u_1^i, \dots, u_{p_i}^i, v_1^i, \dots, v_{p_i}^i$ the vertices in the block Pr_{p_i} , $c = v_1^i$ the terminal vertex, and Pr_{p_i-1} the subgraph obtained by deleting c from the block Pr_{p_i} .

The following observations are needed in determining the metric dimension of vertex-amalgamation of prisms.

Observation 2. *If R is a resolving set of Vertex - Amal $\{Pr_{p_i}; v_1^i\}$ then $|Pr_{p_i} \cap R| \geq 1$ for all i .*

Proof. Suppose that there exists j such that $Pr_{p_j} \cap R = \emptyset$ then $r(v_2^j|R) = r(v_{p_j}^j|R)$, a contradiction. \square

Observation 3. Let R be a resolving set of $Vertex - Amal\{Pr_{p_i}; v_1^i\}$. If p_i is even then $|Pr_{p_i} \cap R| \geq 2$.

Proof. By Observation 2, $|Pr_{p_i} \cap R| \geq 1$. Suppose that $|Pr_{p_i} \cap R| = 1$ and x is the vertex in $Pr_{p_i} \cap R$. If $x \in \{u_1^i, u_{\frac{p_i}{2}+1}^i, v_{\frac{p_i}{2}+1}^i\}$ then $r(u_2^i|R) = r(u_{p_i}^i|R)$. If $x \in \{u_{\frac{p_i}{2}+2}^i, \dots, u_{p_i}^i, v_2^i, \dots, v_{\frac{p_i}{2}}^i\}$ then $r(u_1^i|R) = r(v_{p_i}^i|R)$. If $x \in \{u_2^i, \dots, u_{\frac{p_i}{2}}^i, v_{\frac{p_i}{2}+2}^i, \dots, v_{p_i}^i\}$ then $r(u_1^i|R) = r(v_2^i|R)$. All possible cases lead to contradiction and so $|Pr_{p_i} \cap R| \geq 2$. \square

Observation 4. Let R be a resolving set of $Vertex - Amal\{Pr_{p_i}; v_1^i\}$. If p_i and p_j are both odd then $|(Pr_{p_i} \cup Pr_{p_j}) \cap R| \geq 3$.

Proof. By Observation 2, $|Pr_{p_i} \cup Pr_{p_j} \cap R| \geq 2$. Suppose that $|Pr_{p_i} \cup Pr_{p_j} \cap R| = 2$ and x is the vertex in $Pr_{p_i} \cap R$. If $x = u_1^i$ then $r(u_2^i|R) = r(u_{p_i}^i|R)$. If $x \in \{u_{\frac{p_i+3}{2}}^i, \dots, u_{p_i}^i, v_2^i, \dots, v_{\frac{p_i+1}{2}}^i\}$ then $r(u_1^i|R) = r(v_{p_i}^i|R)$. If $x \in \{u_2^i, \dots, u_{\frac{p_i+1}{2}}^i, v_{\frac{p_i+3}{2}}^i, \dots, v_{p_i}^i\}$ then $r(u_1^i|R) = r(v_2^i|R)$. Thus we have $|(Pr_{p_i} \cup Pr_{p_j}) \cap R| \geq 3$. \square

Now we are ready to prove the following.

Theorem 5. Let $\{Pr_{p_1}, Pr_{p_2}, \dots, Pr_{p_n}\}$ be a collection of n prisms with n_o prisms of odd order. If G is the vertex-amalgamation of $Pr_{p_1}, \dots, Pr_{p_n}$ then

$$\dim(G) = \begin{cases} \sum_{i=1}^n \dim(Pr_{p_i}) - n & , n_o = 0, \\ \sum_{i=1}^n \dim(Pr_{p_i}) - n + n_o - 1 & , n_o \geq 1 \end{cases}$$

Proof. For $n_o = 0$, we have $\dim(G) \geq 2n$ (by Observation 3). Now, we define $S = \bigcup_{i=1}^n \{u_{\frac{p_i}{2}}^i, v_{\frac{p_i}{2}}^i\}$. It is clear that if x, y are two distinct vertices in Pr_{p_i-1} with $d(x, u_{\frac{p_i}{2}}^i) = d(y, u_{\frac{p_i}{2}}^i)$ and $d(x, v_{\frac{p_i}{2}}^i) = d(y, v_{\frac{p_i}{2}}^i)$ then $d(x, c) \neq d(y, c)$. This leads to S being a resolving set and so $\dim(G) = 2n = \sum_{i=1}^n \dim(Pr_{p_i}) - n$.

For $n_o \geq 1$, by applying Observation 3, we have each Pr_{p_i} with even p_i contains at least 2 vertices in a resolving set and, by applying Observation 4, we have each Pr_{p_i} with odd p_i contains at least 2 vertices in a resolving set, except for exactly one which contains only 1 vertex. Therefore $\dim(G) \geq 2(n - n_o) + 2n_o - 1 = 2n - 1$. For the upper bound, we denote by p_{i_o} the minimum among the odd p_i s. Define $S = \bigcup_{i \neq i_o} \{u_{\lceil \frac{p_i}{2} \rceil}^i, v_{\lceil \frac{p_i}{2} \rceil}^i\} \cup \{v_{\lceil \frac{p_{i_o}}{2} \rceil}^{i_o}\}$. It is a routine exercise to show that S is a resolving set and we obtain $\dim(G) = 2n - 1 = \sum_{i=1}^n \dim(Pr_{p_i}) - n + n_o - 1$. \square

Consider edge-amalgamation of $\{Pr_{p_1}, Pr_{p_2}, \dots, Pr_{p_n}\}$ a finite collection of prisms. We denote by $u_1^i, \dots, u_{p_i}^i, v_1^i, \dots, v_{p_i}^i$ the vertices in the block Pr_{p_i} , $c_1c_2 = v_1^i v_{p_i}^i$ the terminal edge, and Pr_{p_i-2} the subgraph obtained by deleting c_1c_2 from the block Pr_{p_i} . The following observations are essential and can be proved similarly to those of vertex-amalgamation of prisms.

Observation 5. *If R is a resolving set of Edge-Amal $\{Pr_{p_i}; v_1^i v_{p_i}^i\}$ then $|Pr_{p_i} \cap R| \geq 1$ for all i .*

Observation 6. *If R is a resolving set of Edge-Amal $\{Pr_{p_i}; v_1^i v_{p_i}^i\}$ then $|(Pr_{p_i} \cup Pr_{p_j}) \cap R| \geq 3$ for all distinct p_i and p_j .*

Now we are ready to prove the last theorem.

Theorem 6. *Let $\{Pr_{p_1}, Pr_{p_2}, \dots, Pr_{p_n}\}$ be a collection of n prisms with n_o prisms of odd order. If G is the edge-amalgamation of $Pr_{p_1}, \dots, Pr_{p_n}$ then*

$$\dim(G) = \sum_{i=1}^n \dim(Pr_{p_i}) - n + n_o - 1.$$

Proof. By Observation 6, we have $\dim(G) \geq 2n - 1$. Now, we denote by p_{i_o} the minimum among the odd p_i s and define

$$S = \bigcup_{\text{even } p_i} \{u_{\frac{p_i}{2}}^i, v_{\frac{p_i}{2}}^i\} \bigcup_{\text{odd } p_i, i \neq i_o} \{v_{\frac{p_i+1}{2}}^i, u_1^i\} \bigcup \{v_{\frac{p_{i_o}+1}{2}}^{i_o}\}.$$

It can be checked that S is a resolving set and so $\dim(G) = 2n - 1 = \sum_{i=1}^n \dim(Pr_{p_i}) - n + n_o - 1$. \square

The afore-mentioned results for complete graphs and prisms arise to the following more general questions.

Open Problem 1. *Let $\{G_1, G_2, \dots, G_n\}$ be a finite collection of graphs and v_{0_i} is a terminal vertex of G_i , $i = 1, 2, \dots, n$. Determine $\dim(\text{Vertex-Amal}\{G_i; v_{0_i}\})$ in terms of $\dim(G_i)$ s.*

Open Problem 2. *Let $\{G_1, G_2, \dots, G_n\}$ be a finite collection of graphs and e_{0_i} is a terminal edge of G_i , $i = 1, 2, \dots, n$. Determine $\dim(\text{Edge-Amal}\{G_i; e_{0_i}\})$ in terms of $\dim(G_i)$ s.*

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